Foam Consolidation and Drainage

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ABSTRACT: A theoretical model of foam as a consolidating continuum is proposed. The general model is applied to foam in a gravity settler. It is predicted that liquid drainage from foam in a gravity settler begins with a slow drainage stage. Next, a stage with faster drainage occurs where the drainage rate doubles compared to the initial stage. The experiments conducted within the framework of this work confirmed the theoretical predictions and allowed measurements of foam characteristics. Foams of three different concentrations of Pantene Pro-V Classic Care Solutions shampoo were studied, as well as the addition of polyethylene oxide (PEO) in one case. The shampoo’s main foaming components are sodium lauryl sulfate and sodium laureth sulfate. It is shown to what extent foam drainage is slowed down by using higher shampoo concentrations and how it is further decreased by adding polymer (PEO).

1. INTRODUCTION

The theoretical and experimental investigations of foams were in focus due to their multiple applications in material processing and the pharmaceutical and food industries.1,2 Multiple investigations of foams differ by the method of foam creation and measurement technique. One way of creating foam is to pass a gas through an aqueous solution of surfactant. For example, the authors of refs 3 and 4 generated foam by bubbling nitrogen through their solution. Since bubbling gas through a solution takes time to fill a column with foam, drainage from the first bubble layer would begin as soon as it is created, making foam evolution less uniform. On the other hand, foams created through mechanical agitation can be expected to have a more uniform distribution of liquid compared to the foams generated by bubbling a gas through surfactant solutions.5 In ref 6, a high speed mixing chamber was used in which the surfactant solution and gas were mixed to create foam. One of the major advantages of this method is that large amounts of uniform foam can be created.

Once foam is formed, drainage characteristics can be investigated. In particular, the following parameters can be determined: (i) the velocity of liquid drainage from the foam, (ii) the percolation velocity of liquid flow in the foam, (iii) foam pressure drop, (iv) forced drainage, and (v) free drainage.7

In the present work, free drainage of foam was investigated. Different measurement techniques have been devised to measure the liquid content in foam during drainage.3,4,8 In particular, to find the liquid fraction the authors of ref 8 measured the transmission speed of sound waves from a speaker to a microphone. In ref 9, the inside of a foam-containing column was vertically lined with wires allowing AC conductivity measurements to find the liquid fraction. Lastly, the authors of ref 10 used nuclear magnetic resonance imaging (NMRI) to find the liquid fraction and then were able to calculate the liquid drained. Their technique was sensitive enough that they developed a method to find the amount of excess liquid drained due to bubble breakage.

When measuring foam drainage, it was found by different authors that only certain factors influence the drainage rate and amount drained, although there are disagreements. For example, the authors of ref 11 found that different gases used in creating foams can change the drainage rate and overall drainage time. In ref 3, it was concluded that foam drainage was faster and the amount of liquid drained was larger for smaller bubbles and larger initial heights, whereas in ref 4, it was stated that the drainage rate was slower for larger initial heights and the maximal rate was the same regardless of height. However, factors such as column diameter and column material have no effect on the rate or drainage time according to ref 4. In addition, in ref 8, it was found that the time to full drainage was inversely related to the liquid fraction.

Several empirical equations were proposed to fit foam drainage data. One of the first empirical foam drainage equations was proposed in ref 12. This equation relates the fraction of liquid in the foam to an exponential function involving the current and characteristic time, with the latter being a fitting parameter. Its popularity is probably the result of its simplicity and sufficient accuracy. An equation that predicts the liquid drained as a function of time was also proposed in ref...
4. However, in addition to the liquid volume fraction, this equation involves such parameters as bubble diameter, solution viscosity, radius of Plateau border,\(^1\) and the lamellae thickness in the foam, which cannot be easily established in ordinary foams.

In the present work, foam is considered in the framework of consolidation theory (section 2) and the predictions made are compared to our experiments of foam drainage in a gravity settler, described in section 3. The discussion is given in section 4. Conclusions are drawn in section 5.

2. THEORETICAL

2.1. General Equations. According to ref 1, foam in which gas bubbles have already crowded, begun to collide, and press on each other significantly possess elasticity. The bubbles and surfactant-stabilized liquid lamellae in foam form an elastic skeleton. In addition, solvent drains through the lamellae driven by a pressure gradient according to Darcy’s law

\[
v_{LS} = v^L - v^S = -\frac{k}{\mu} \nabla p
\]

(1)

where \(v^L\), \(v^S\), and \(v_{LS}\) are the liquid and skeleton velocities and the liquid-relative-to-skeleton average velocity, respectively, \(k\) is the skeleton permeability on the order of \(k = \delta^2/12\) with \(\delta\) being the average lamella thickness, \(\mu\) is the solvent viscosity, and \(p\) is the solvent pressure.

Foam, as a system, comprised of an elastic skeleton assumed to follow Hooke’s law with viscous liquid drainage in the lamellae perfectly resembles the medium dealt with by consolidation theory of soil mechanics\(^{13,14}\) and is a medium lamellae perfectly resembles the medium dealt with by consolidation theory (section 2) and the predictions made are compared to our experiments of foam drainage in a gravity settler, described in section 3. The discussion is given in section 4. Conclusions are drawn in section 5.

\[
\nabla \cdot \sigma' = \frac{1}{2} \frac{E}{(1 + \nu)(1 - 2\nu)} \left[ \nabla^2 u + \frac{1}{1 - 2\nu} \nabla (\nabla \cdot u) \right]
\]

(8)

Equations 2 and 3 and combined eqs 5 and 8 form a system of two equations for the two unknowns \(\nu\) and \(\mu\), namely

\[
\frac{\partial (\nabla \cdot u)}{\partial t} = \frac{k}{\mu} \nabla^2 p
\]

(9)

Equation 6 allows one to find, as in ref 16, that

\[
\nabla \cdot \sigma' = \frac{1}{2} \frac{E}{(1 + \nu)(1 - 2\nu)} \left[ \nabla^2 u + \frac{1}{1 - 2\nu} \nabla (\nabla \cdot u) \right] = \nabla p
\]

(10)

Taking the divergence of eq 10 yields

\[
\frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \nabla^2 (\nabla \cdot u) = \nabla^2 p
\]

(11)

Equation 9 allows us to exclude \(p\) from eq 11, which yields the following scalar equation containing solely \(u\), in particular, \(\varepsilon_v = \nabla \cdot u\),

\[
\frac{\partial \varepsilon_v}{\partial t} = c_v \nabla^2 \varepsilon_v
\]

(12)

where

\[
c_v = \frac{k}{\mu (1 + \nu)(1 - 2\nu)} \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)}
\]

(13)

Equation 12 describes the evolution of the volumetric strain during skeleton consolidation accompanying liquid drainage. It coincides formally with the standard parabolic heat transfer equation, where the role of the transport coefficient, in the present case, is played by the consolidation coefficient\(^{14}\) \(c_v\) given by eq 13.

In addition, eq 11 can be integrated, which yields the following relation between \(p\) and \(\varepsilon_v\)

\[
p = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \varepsilon_v + f(r, t)
\]

(14)
where $F$ is any harmonic function of the position vector $\mathbf{r}$, which can also contain time as a parameter.

2.2. Gravity Settler. In an axisymmetric cylindrical gravity settler initially filled with uniform foam up to height $h$, a new physical factor, namely gravity, plays role and liquid drainage to the bottom is anticipated (Figure 1).

![Figure 1. Filled cylinder of foam with surfactant solution (drained liquid) below. The top of the foam is at $z = 0$, and the bottom of the cylinder is at $z = -h$.](image)

In the presence of gravity, Darcy’s law (1) is amended and takes the following form:

$$
\nu^{LS} = \nu^L - \nu^S = -k \nabla (p + \rho g z)
$$

(15)

where $\rho$ is the liquid density, $g$ is the acceleration due to gravity, and $z$ is the vertical coordinate with $z = 0$ corresponding to the free surface and $z = -h$ to the bottom. Equation 15 expresses the fact that the hydrostatic pressure distribution in the lamellae at rest ($p = p_0 - \rho g z$ with $z$ being negative and $p_0$ being atmospheric pressure), should not result in any drainage against gravity, as is physically expected.

The presence of the gravity-related term in eq 15 does not preclude arriving at eq 2 by the previous derivation, since $V^2 z \equiv 0$. Also, eq 5 which expresses the balance of the elastic stresses due to the surfactant and liquid pressure does not change. Then eqs (8–14) also do not change. In particular, eqs 12 and 14 in the present case reduce to the following equations

$$
\frac{\partial \nu}{\partial t} = \nu \frac{\partial^2 \nu}{\partial z^2}
$$

(16)

$$
p = \frac{E(1 - \nu)}{(1 + \nu)(1 - 2\nu)} \epsilon_v + Az + B
$$

(17)

where $A$ and $B$ are constants.

Substituting eq 17 into eq 16, we obtain the following equation for the liquid pressure

$$
\frac{\partial p}{\partial t} = \nu \frac{\partial^2 p}{\partial z^2}
$$

(18)

The drainage velocity $v$ in the gravity direction according to eq 15 is given by

$$
v = -\frac{k}{\mu} \frac{\partial (p + \rho g z)}{\partial z}
$$

(19)

The drainage velocity $v$ is equal to zero at the bottom (at $z = -h$). It can also be assumed to equal zero at the free surface, so that the boundary conditions for eq 18 at $t > 0$ read

$$
\frac{\partial p}{\partial z} \bigg|_{z=0 or z=-h} = -\rho g
$$

(20)

The boundary condition (20) at $z = 0$ implies that foam is pinned at the column top where $z = 0$, which is indeed the case as discussed below. It is also emphasized that in the experiments of section 3, the drained (unfoamed) liquid column accumulates near the settler bottom in the course of drainage and the lower boundary of foam moves from $z = -h$ up. However, during the entire drainage process, the height of this drained liquid column is less than 7 cm, whereas the foam column height is 104 cm. Therefore, in the first approximation, dealing with foams with the initial gas volume fraction $\phi$ of about 0.95 (Table 1), we neglect the effect of motion of the lower foam boundary and displace the boundary condition (20) to $z = -h$ for any $t$.

The initial condition corresponds to uniform foam

$$
p|_{t=0} = p_0
$$

(21)

Equations 18 and 20 show that the problem has an integral invariant

$$
\int_{-h}^{0} p \, dz = p_0 h = \text{const}
$$

(22)

which can be recast with the help of eq 17 into

$$
\int_{-h}^{0} \epsilon_v \, dz = \left(1 + \nu(1 - 2\nu)\right) \left(p_0 h + \frac{Ak^2}{2} - Bh\right)
$$

(23)

Table 1. Results for Different Concentrations of Shampoo and PEO

<table>
<thead>
<tr>
<th>concentration</th>
<th>2% shampoo</th>
<th>4% shampoo</th>
<th>8% shampoo</th>
<th>8% shampoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>trial number</td>
<td>1 2 1 2 1 2</td>
<td>1 2 1 2</td>
<td>1 2 1 2</td>
<td>1 2 1 2</td>
</tr>
<tr>
<td>minimum slope $\times 10^3$ (cm/s)</td>
<td>15.0 14.6</td>
<td>9.4 11.3</td>
<td>9.1 9.8</td>
<td>4.9 6.3</td>
</tr>
<tr>
<td>maximum slope $\times 10^3$ (cm/s)</td>
<td>30.5 29.3</td>
<td>20.9 23.0</td>
<td>21.2 21.3</td>
<td>13.6 14.4</td>
</tr>
<tr>
<td>max. slope/min. slope</td>
<td>2.03 2.01</td>
<td>2.22 2.04</td>
<td>2.33 2.17</td>
<td>2.78 2.29</td>
</tr>
<tr>
<td>average permeability, $k \times 10^5$ (cm$^2$)</td>
<td>1.54 1.49</td>
<td>1.01 1.16</td>
<td>1.00 1.04</td>
<td>2.03 2.34</td>
</tr>
<tr>
<td>consolidation coefficient, $c_v$ (cm$^2$/s)</td>
<td>2.57 2.57</td>
<td>1.90 1.96</td>
<td>1.64 1.89</td>
<td>1.16 1.24</td>
</tr>
<tr>
<td>initial gas volume fraction, $\phi$</td>
<td>0.943 0.942</td>
<td>0.949 0.945</td>
<td>0.953 0.950</td>
<td>0.955 0.951</td>
</tr>
<tr>
<td>Young’s modulus [eq 7], E, GPa</td>
<td>0.29 0.29</td>
<td>0.29 0.29</td>
<td>0.29 0.29</td>
<td>0.29 0.29</td>
</tr>
<tr>
<td>Young’s modulus [eq 13], E, GPa</td>
<td>0.17 0.17</td>
<td>0.19 0.17</td>
<td>0.16 0.18</td>
<td>0.19 0.18</td>
</tr>
</tbody>
</table>
It is easy to see that the left-hand side of eq 23 corresponds to nothing but the liquid content in the foam layer $-h \leq z \leq 0$, which obviously should not change in time. Indeed, according to eq 23, it does not.

It is convenient to introduce $P = p + \rho gz$, and render $t$ dimensionless by $h^2/\varepsilon_c; p, p_0$, and $P$ by $\rho gh$; and $z$ by $h$. Then, the problem described by eqs 18, 20, and 21 can be recast to the following dimensionless form

$$\frac{\partial P}{\partial t} = \frac{\partial^2 P}{\partial z^2} \quad (24)$$

$$\frac{\partial P}{\partial z} |_{z=0; \text{or} \ z=-1} = 0 \quad \text{at} \quad t > 0 \quad (25)$$

$$P = p_0(z) = p_0 + z \quad \text{at} \quad t = 0 \quad (26)$$

where bars over the dimensionless parameters are omitted for brevity.

The integral invariant (22) takes the form

$$\int_{-1}^{0} P \, dz = p_0 - \frac{1}{2} \quad (27)$$

corresponding to eqs 24–26.

Finding the solution of the problem 24–26 and returning from $P$ to the dimensionless pressure $p$, one obtains

$$p = p_0 - \sum_{n_{\text{odd}} = 1}^{\infty} \frac{4 \pi^2}{3 \pi^2} \left[ 1 - \exp(-\pi^2 n^2 t) \right] \cos(\pi nz) \quad (28)$$

It is easy to see that as $t \to \infty$, according to eq 28, $p \to p_0 = p_0 - z - 1/2$. In dimensional units, this means that the pressure distribution tends to

$$p = p_0 - \frac{\rho gh}{2} - \rho gz \quad (29)$$

As a result, one expects that, at the pinned free surface $z = 0$ (i.e., the free surface of foam which cannot slip down the settler wall during foam drainage), pressure will become lower than the atmospheric pressure, namely, $p_0 - \rho gh/2$. The difference is small, $\rho gh/2 \approx 0.01 p_0$, and represents itself the price of the assumption that the free surface is pinned. In reality, the free surface was pinned at the settler wall and cap.

Using solution 28 and eq 19, we find the dimensionless convectional velocity in the following form (albeit $z$ is still dimensionless)

$$v = -\frac{k p g}{\mu} \left\{ \sum_{n_{\text{odd}} = 1}^{\infty} \frac{2}{n \pi} (1 - \exp(-\pi^2 n^2 t)) \sin(\pi nz) + 1 \right\} \quad (30)$$

Equation 30 shows that at $t = 0$ at any $z$ (in particular at the settler bottom) the drainage velocity $v = -k p g/\mu$. At time $t > h^2/(\varepsilon_c)$, the drainage velocity at any $z$ doubles, $v = -2k p g/\mu$.

Foam in the settler does not detach from the walls. Therefore, in the present case, $u_{xx} = u_{yy} = 0$, while $v_{xx} = u_{zz}$. Then, according to eq 6, we find in the present case

$$\sigma_{zz} = \left( \frac{E \varepsilon_p (1 - \nu)}{(1 + \nu)(1 - 2\nu)} - p \right) \quad (31)$$

whereas the lateral stresses $\sigma_{xx}$ and $\sigma_{yy}$ are nonzero, as is expected in the laterally restricted situation.

Using eq 17, it is easy to see that

$$\varepsilon_v = \left( \frac{1 + \nu}{(1 - 2\nu)} \right) \sum_{n_{\text{odd}} = 1}^{\infty} \frac{4 \pi^2}{\pi^2 n^2} \exp(-\pi^2 n^2 t) \cos(\pi nz) \quad (32)$$

with $t$ and $z$ being dimensionless.

Then, from eqs 31, 32, and 28 we find that the stress, rendered dimensionless by $\rho gh$, is given by

$$\sigma_{zz} = -p_0 - \sum_{n_{\text{odd}} = 1}^{\infty} \frac{4 \pi^2}{\pi^2 n^2} (2 - \exp(-\pi^2 n^2 t)) \cos(\pi nz) \quad (33)$$

This equation shows that the stress on the bubbles near the bottom is always compressive (negative) and its magnitude increases in time from the dimensionless value of $(p_0 + 1/2)$ to $(p_0 + 1)$. The experiments show that the increasing pressure near the settler bottom leads to breakup of larger bubbles and formation of the smaller ones. This, however, cannot change the effective Young’s modulus $E$ and Poisson’s ratio $\nu$ of the foam if no gas is lost, i.e. $\varphi = \text{const}$ [cf. eqs 7].

It is emphasized that the above theory assumes that the lamellae thicknesses in the foam are sufficiently large to neglect the effect of the long-range forces (e.g., the van der Waals forces) associated with the disjoining pressure. This assumption is sufficiently accurate for the lamellae thicknesses above $1 \mu$m. However, at the later stage of liquid drainage from the foam the lamellae can thicken below this threshold and the disjoining pressure tends to stabilize them, which decelerates the latest stage of the drainage process. This effect will also be visible in the experimental results described in section 4, and the present theory cannot be applied to the latest stage of drainage.

3. EXPERIMENTAL SECTION

In order to create a large amount of uniform foam, the mechanical mixing method was chosen over the blowing bubbles method. To create the surfactant solution, Pantene Pro-V Classic Care Solutions shampoo (store bought and mostly containing sodium lauryl sulfate and sodium laureth sulfate) and DI water were placed into a mixing bowl in various proportions to produce different concentrations by volume. Poly(ethylene oxide), PEO, of $M_n = 8$ MDa was also added to the surfactant solution in one case. PEO was purchased from Sigma Aldrich. A standard household mixer, Hamilton Beach, stirred the solution on the highest setting for 2 min except when PEO was added. In the latter case, PEO was added to the DI water and mixed for 2 min, then the shampoo was added and stirred again for 2 min. It is emphasized that a wire impeller was used because of its ability to create a more uniform bubble size and better mixed foam (no excess water) than the standard propeller. The foam was then poured into a squeezable funnel with a flexible hose (inside diameter was 1.7 cm) attached to the end. The funnel was squeezed to force the foam through the hose. The cylinder of the gravity settler was filled from the bottom up as shown in Figure 2. As the level of foam increased, the funnel–hose system was raised correspondingly so that the foam exiting the hose would be at the top. When the foam was filled to the top of the cylinder, the funnel–hose system was removed and the cylinder was capped, which effectively prevented loss of liquid vapor or gas from the settler during the entire experiment. The time to fill the cylinder with foam averaged about 1 min, during which no drained liquid was visible. The inside dimensions of the cylinder were 2.58 cm.
in diameter and 104 cm tall. The drained liquid height was measured using a CCD camera and a backlight shown in Figure 3.

4. RESULTS AND DISCUSSION

The present work aims at measuring the amount of liquid drained as well as the rate of drainage as functions of time. The height of the drained liquid/foam interface was measured by counting pixels in the images taken by the CCD camera and using the outside cylinder width as a reference scale. Each pixel corresponded to approximately 0.13 mm. Several images taken during drainage of liquid in an experiment with 2% shampoo foam are shown in Figure 4. The dashed line indicates the level of the drained liquid from the previous image. Figure 4a corresponds to the first instant when drained liquid appears 1.5 min after the column was capped. The progressive images show how the rate of drainage initially increases from Figure 4a to d and then decreases from Figure 4e to h.

The evolution of the drainage rate was found to be in agreement with the theoretical predictions of section 2. When drainage begins, its initial rate is relatively slow and is approximately constant (Figure 5). Next, the drainage rate increases in time until a maximum is reached, corresponding to twice the initial drainage rate, and proceeds at this constant higher rate (Figure 5). Figure 5 also shows that the drainage process begins to slow down compared to the maximum rate of drainage at $t > 800$ s. This phenomenon is probably characteristic of older and/or dryer foam, in which the lamellae thickness is below 1 μm, and the disjoining pressure (not accounted for in the present theory) decelerates the drainage process.

Figure 6 shows that an increase in shampoo concentration from 2% to 4% slows the drainage process. However, a further increase from 4% to 8% results in an insignificant change in the drainage duration or rates. This is due to the fact that the critical micelle concentration (cmc) of surfactant is being reached between 2% and 4%. Indeed, the main surfactant in the shampoo is sodium lauryl sulfate which has a cmc of 0.243%.19,20 The water content in 2% shampoo is close to 99.6%. That means that the remaining 0.4% is composed of...
calculated for the slow and fast drainage processes. They were measured and listed in Table 2. The values of (green) 8% shampoo, and (purple) 8% shampoo and 0.05% PEO. consistent between trials: (blue) 2% shampoo, (red) 4% shampoo, concentration, the result of one trial is shown because the data are taken as normalized by the total volume of liquid in the foam, \( V \). For each concentration, the result of one trial is shown because the data are consistent between trials: (blue) 2% shampoo, (red) 4% shampoo, (green) 8% shampoo, and (purple) 8% shampoo and 0.05% PEO.

Figure 6. Drainage curves for different concentrations of shampoo (and addition of PEO). The volume of liquid in the foam, \( V \), is normalized by the total volume of liquid in the foam, \( V_0 \). For each concentration, the result of one trial is shown because the data are consistent between trials: (blue) 2% shampoo, (red) 4% shampoo, (green) 8% shampoo, and (purple) 8% shampoo and 0.05% PEO.

Young’s modulus results only from the gas inside the bubbles. The Young’s modulus was calculated using eq 13. A fairly good agreement between the values of Young’s modulus found by the two methods can be seen in Table 1. It is instructive to see that Young’s modulus does not depend on the surfactant.

Table 1 represent their averages. The permeability values approximately ranged between \( 1 \times 10^{-7} \) and \( 2.4 \times 10^{-7} \) cm².

Foam permeability decreases when shampoo concentration increases, which corresponds to slowing down of the drainage process at higher shampoo concentration. Another way to evaluate the permeability is to employ direct observations under a microscope of the liquid lamellae in the foam and consider them as planar channels supporting Poiseuille-like drainage flow. For planar Poiseuille flow, the permeability is equal to \( k = \frac{\delta^2}{12} \) where \( \delta \) is the channel (lamella) width (thickness), which follows from the Poiseuille law written in the form of Darcy’s law. For all concentrations of shampoo with no PEO added, the lamellae thickness \( \delta \) in fresh foam ranges from 20 to 50 \( \mu \)m, as measured using the foam images. Therefore, the permeability value estimates for the Poiseuille-like channel flow range from \( 3.3 \times 10^{-7} \) to \( 2.1 \times 10^{-6} \) cm², respectively. The value of \( k \) of the order of \( 10^{-7} \) cm² estimated assuming the Poiseuille-like drainage flow overlaps with the data in Table 1 found from the drainage rates. Therefore, the two separate estimates of the permeability are in a reasonable agreement with each other. It is emphasized that drainage through an individual lamella happens in the direction of its generatrix, which may be inclined relative to the \( z \)-axis. On the other hand, the cumulative drainage from the foam happens parallel to the \( z \)-axis [downward, cf. eq 19]. The result obtained shows that the average permeability of the individual lamellae is of the same order as the foam permeability as a whole.

Also presented in Table 1 are the values of the consolidation coefficient and the gas volume fractions. The consolidation coefficient was calculated using the intersection time \( t_i \) between the slopes of the slow and fast drainage processes (cf. Figure 5) and the value of the column height using the equation \( c = \frac{h^2}{\pi t} \). The consolidation coefficient decreases as shampoo concentration increases. Adding PEO decreases the consolidation coefficient even further. Lower values of the consolidation coefficient correspond to higher foam stability.

The gas volume fraction, \( \varphi \), is a given parameter; its values are listed in Table 1. Using eq 7, Poisson’s ratio, \( \nu \), is calculated. The values of \( \nu \) are in the range from 0.02 to 0.03.

The Young’s modulus, \( E \), was calculated using two different ways and the values are compared, as shown in Table 1. First, the Young’s modulus was calculated using eq 7 where it is related to atmospheric (gas) pressure in the bubbles under the assumption that capillary pressure is negligible and the foam elasticity results only from the gas inside the bubbles. The second way to calculate the Young’s modulus is using its relation to the consolidation coefficient, as in eq 13. Using the values of the consolidation coefficient, average permeability, Poisson’s ratio, and liquid viscosity from Tables 1 and 2, Young’s modulus was calculated from eq 13. A fairly good agreement between the values of Young’s modulus found by the two methods can be seen in Table 1. It is instructive to see that Young’s modulus does not depend on the surfactant.

Table 2. Viscosity for Different Concentrations of Shampoo and PEO in Unfoamed Solutions

<table>
<thead>
<tr>
<th>Concentration</th>
<th>2% Shampoo</th>
<th>4% Shampoo</th>
<th>8% Shampoo</th>
<th>8% and 0.05% PEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity (cP)</td>
<td>1.0</td>
<td>1.0</td>
<td>1.1</td>
<td>3.4</td>
</tr>
</tbody>
</table>

*Measurements were conducted using a Brookfield cone and plate viscometer.
concentration, which corroborates the idea that foam elasticity is associated with the gas entrapped in the bubbles. To ensure complete mixing for viscosity measurements, solutions were prepared and stirred for an appropriate time. Then, a bottle with a solution was capped and left overnight to allow the foam to settle. The drained fluid was the bulk solution from the foam lamellae. All the aqueous shampoo solutions were found to be Newtonian. On the other hand, the shampoo solution with added PEO was slightly shear-thinning. Its viscosity decreased from 3.8 to 3.0 cP when the shear rate increased from 37.5 to 375 s$^{-1}$. Since the change in viscosity is small, the average value listed in Table 2 was used in all calculations.

It is also of interest to compare our data to the empirical formula for the volume fraction of liquid proposed in ref 12.

$$\frac{V}{V_0} = \exp(-Kt)$$  \hspace{1cm} (34)

where $V$ is the volume of liquid in the foam, $V_0$ is the initial liquid volume in the foam as in Table 3, $t$ is time, and $K$ is an empirical constant (the inverse characteristic drainage time, which corresponds to diminishing of the liquid volume in the foam to about 1/3 of its initial value, and is to be found as a result of curve fitting). Equation 34 is applied to the second half of our data in Figure 6, as suggested in ref 22. The average (per each foam composition) values of $K$ are 0.001 06, 0.000 80, 0.000 80, and 0.000 60 s$^{-1}$ for 2%, 4%, 8%, and 8% shampoo with 0.05% PEO, respectively. It is clearly seen in Figure 7a–d that eq 34 is incapable of describing the initial (slow) drainage stage and the maximum drainage stage, even though it can fit the rest of the dependence with the appropriate choice of the fitting parameter $K$.

5. CONCLUSION

The present theory treats foam as a consolidating medium possessing elastic properties predominantly due to gas present in the bubbles, as well as a Darcy-like drainage of liquid from their interconnected lamellae. It is predicted that the drainage process begins with a slow stage. This stage lasts on the scale of the characteristic consolidation time. After that, it is replaced by a faster drainage stage in which the rate of drainage doubles. This prediction was confirmed by the current experiments with foams with different surfactant concentrations and a polymer additive. At the latest stage of foam existence, the drainage slowdown could be attributed to a deceleration by the disjoining pressure in liquid lamellae. The experiments also showed that the foam drainage rate in a gravity settler is diminished at higher surfactant concentration to a certain limit.
of approximately 4% for this surfactant, and beyond the limit with the addition of a small amount of PEO. As predicted, the effective Young’s modulus of foam appeared to be determined by the gas present in the bubbles and is practically independent of surfactant concentration. Overall, the experimental results confirmed the main elements of the theory. The general theory of foam as a consolidating medium can be applied to other situations.

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Notes

The authors declare no competing financial interest.

■ ACKNOWLEDGMENTS

This work was supported by The US Gypsum Corporation (USG). The authors thank Dr. K. Natesaiyer for useful discussions.

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